

## 1 Graph Representations

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Write the graph above as an adjacency matrix, then as an adjacency list.

## 2 DFS and BFS

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Give the DFS preorder, DFS postorder, and BFS order of the graph starting from vertex *A*. Break ties alphabetically.

## 3 Topological Sorting

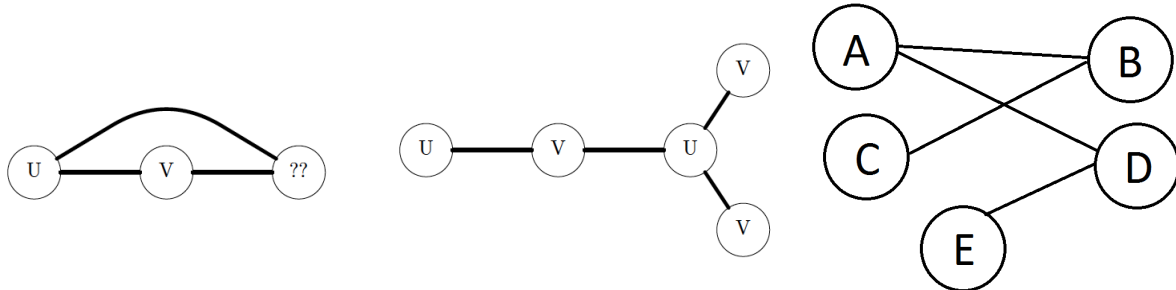
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Give a valid topological sort of the graph above. (Hint: Use the reverse postorder.)

## 4 Graph Algorithm Design: Bipartite Graphs

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An undirected graph is said to be bipartite if all of its vertices can be divided into two disjoint sets  $U$  and  $V$  such that every edge connects an item in  $U$  to an item in  $V$ . For example, the graphs in the center and on the right are bipartite, whereas the graph on the left is not. Provide an algorithm which determines whether or not a graph is bipartite. What is the runtime of your algorithm?



## 5 Extra for Experts: Shortest Directed Cycles

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Provide an algorithm that finds the shortest directed cycle in a graph in  $O(EV)$  time and  $O(E)$  space, assuming  $E > V$ .

## 6 Extra for Experts: DFS Gone Wrong

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Consider the following implementation of DFS, which contains a crucial error:

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create the fringe, which is an empty Stack
  push the start vertex onto the fringe and mark it
  while the fringe is not empty:
    pop a vertex off the fringe and visit it
    for each neighbor of the vertex:
      if neighbor not marked:
        push neighbor onto the fringe
        mark neighbor
```

Give an example of a graph where this algorithm may not traverse in DFS order.