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## 1 Which is faster?

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For each example below, there are two algorithms solving the same problem. Given the asymptotic runtimes for each, is one of the algorithms **guaranteed** to be faster? If so, which? And if neither is always faster, explain why. Assume the algorithms have very large input (so  $N$  is very large).

A. Algorithm 1:  $\Theta(N)$ , Algorithm 2:  $\Theta(N^2)$

B. Algorithm 1:  $\Omega(N)$ , Algorithm 2:  $\Omega(N^2)$

C. Algorithm 1:  $O(N)$ , Algorithm 2:  $O(N^2)$

D. Algorithm 1:  $\Theta(N^2)$ , Algorithm 2:  $O(\log N)$

E. Algorithm 1:  $O(N \log N)$ , Algorithm 2:  $\Omega(N \log N)$

Would your answers above change if we did not assume that  $N$  was very large?

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## 2 More Runtime Analyzing

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A. How many times is `lobsterPainting` called? Give your answer in  $\Theta$  notation in terms of  $N$ , assuming `lobsterPainting` does not crash or call any methods.

```
1 for (int i = 0; i < N/2; i++) {
2     for (int j = i - 1; j < N/2 + 1; j++) {
3         lobsterPainting(i, j);
4     }
5 }
```

B. How about here?

```
1 for (int i = N - 1; i > 0; i /= 2) {
2     for (int j = 0; j < i; j++) {
3         lobsterPainting(i, j);
4     }
5 }
```

C. Bonus: And here?

```
1 public static void crabDrawing(int N) {
2     for (int i = 1; i < N; i *= 2) {
3         lobsterPainting(i, i);
4         crabDrawing(i);
5     }
6 }
```

### 3 More? Of Course More

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Describe the best-case and worst-case runtimes of the function individually using  $\Theta$ . Then use them to describe the overall runtime of the function in terms of  $\Theta$  (if possible) or  $O/\Omega$  if not.

A. Assume `arr` is a **sorted** array of **unique** elements of size  $N$ . Example of calling this method would be: `hopps(sortedArr, 0, sortedArr.length)`.

```
1 public static int hopps(int[] arr, int low, int high) {
2     if (high <= low)
3         return -1;
4     int mid = (low + high) / 2; // (later, see http://goo.gl/gQI0FN )
5     if (arr[mid] == mid)
6         return mid;
7     else if (mid > arr[mid])
8         return hopps(arr, mid + 1, high);
9     else
10        return hopps(arr, low, mid);
11 }
```

Bonus: What is `hopps` doing?

B. Assume `str` is a String of characters of size  $N$ .

```
1 public static char wilde(String str) {
2     Map<Character, Integer> map = new HashMap<>();
3     for (char c : str.toCharArray()) {
4         if (map.containsKey(c)) {
5             map.put(c, map.get(c) + 1);
6         } else {
7             map.put(c, 1);
8         }
9     }
10    for (int i = 0; i < str.length(); i++) {
11        if (map.get(str.charAt(i)) == 1) {
12            return str.charAt(i);
13        }
14    }
15    return 0; // 0 represents the NULL character
16 }
```

Bonus: What is `wilde` doing?

Bonus's Bonus: Can you do it with only 1 `for` loop?

## 4 Have You Ever Went Faster? (Extra)

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Given an integer  $x$  and a **sorted** array  $A[]$  of  $N$  distinct integers, design an algorithm to find if there exists distinct indices  $i$ ,  $j$ , and  $k$  such that  $A[i] + A[j] + A[k] == x$ . Feel free to write pseudocode instead of Java. Your code should run in  $\Theta(N^2)$  time.